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June 1980

MECHANICS OF COMPOSITE MATERIALS. SUMMARY OF SOME THEORETICAL AND EXPERIMENTAL STUDIES

(MÉCANIQUE DES MATÉRIAUX COMPOSITES. APERÇUS DE QUELQUES ÉTUDES THÉORIQUES ET EXPÉRIMENTALES)

by

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UDC 621-426 : 539.3/6 : 539.32

ROYAL AIRCRAFT ESTABLISHMENT

Library Translation 2044

Received for printing 26 June 1980

MECHANICS OF COMPOSITE MATERIALS. SUMMARY OF SOME THEORETICAL AND EXPERIMENTAL STUDIES

(MÉCANIQUE DES MATÉRIAUX COMPOSITES. APERÇUS DE QUELQUES ÉTUDES THÉORIQUES ET EXPÉRIMENTALES)

by

T. Vinh

Sciences et Techniques de l'Armement, 52, 1978

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AUTHOR'S SUMMARY

The author has chosen two aspects, namely assessment by calculation and the dynamics of composite materials.

In the first part, the theoretical studies are reviewed, in particular various methods for evaluating elastic constants (boundary methods using variational calculation, methods using Airy stress function, elastodynamic methods) and recent methods of calculation for multilayers in the plastic state. Finally, a critical study of the various theories available for the propagation of waves and vibrations in composite materials is made (equivalent homogeneous media theory, microstructure theory, mixture theory, theory based on asymptotic development, etc).

In the second part, some experimental researches are reviewed for the calculation of elastic constants in micropolar elasticity under a static and dynamic regime. The mechanical characteristics of composite materials are examined along with the corresponding experimental techniques (vibration of rods, ultrasonics, etc).

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1 INTRODUCTION

During the past five years, special composite materials - long fibres, in particular - have been developed for the specific needs of the aeronautical and space industry. In other sectors of industry more and more use is being made of these materials as complements to known metallic materials and plastics. These products are used each time it is desired to blend high mechanical performance with lightness of structure or of mechanical parts.

We are in a phase where ability is developing very fast, and often, in the recent past, it has outstripped knowledge. Around 1960, this situation was an important factor for inciting researchers. It became transformed, very rapidly, by a veritable explosion of studies and researches, some theoretical, others experimental, in all fields. It is sufficient to peruse the technical reviews in applied mechanics to become aware of this activity.

This interest, which borders on infatuation, is surely explained by the desires of engineers and technicians to fill in their basic knowledge, which is often confined to conventional materials. In our opinion it is also explained by the fact that composite materials now stand at a privileged crossroads, to which a large number of scientific disciplines are converging. This situation, unique in the history of materials, is a very happy one and quite rightly favours the development of inter-disciplinary studies, as Fig 1 shows.

In fact, quantum mechanics, which studies periodic lattices, has studies which can be transposed into the field of composite materials[1,2]. Thus, quite recently Kohn *et al*[3] have presented the results of original studies in this field, with a variational formula only slightly known in mechanics. Geology, which is used to treating geological layers in seismology, has studies which are capable of being extrapolated to composite materials. In this field are the works of Jardetzky[4], Brekhovskikh[5] and Postma[6] on the propagation of waves in stratified media.

The physics of metals and viscoelastic materials, together with rheology in general, are, naturally, of great interest in this field, with the focus on new composite materials, such as orientated eutecties and high diffusion fibrous materials. They can give to the composite materials specialists a certain number of experimental results concerning polyphase materials (polycrystalline metals, sequenced polymers, amongst others), as Mason and Skimin[7] show. In return, recent studies on composite materials (periodic or random) could shed a new light on the interpretation of these results (interpretation of multiple peaks of internal friction, wave attenuation, etc).

Acoustics and optics have both developed, in parallel, numerous studies on composite materials. They supply researchers with numerous methods of investigation, ranging from the simplest to the most sophisticated (for example, holographic interferometry). On the other hand, composite materials can form special filters for use in acoustics.

Biomechanics, which is an expanding field, can supply a variety of natural, anisotropic, composite materials (particularly bones) which are impervious or porous. In return, it is not impossible, in the near future, that there will be a development in the use of composite materials, compatible with human tissues, for the fabrication of prosthesis or orthesis items.

Finally, it should be noted that there is a growing [illegible] matics and theoretical mechanics, in composite materials which are either periodic or which have random distribution. During the past ten years there has been an outburst of various studies on such media, in all directions. Mathematical physics methods also find a rich field there.

Taking into account the multiple aspects which composite materials offer, and within the scope of the present, informal report, discussion will be limited to some theoretical and experimental studies.

For the former, two aspects will be examined, assessment calculations and the propagation of waves and vibrations in composite materials. This done, I am very conscious of leaving aside many sectors whose importances are at least equal to those of the chosen areas.

2 ASSESSMENT CALCULATIONS

The purpose of the assessment calculations is to supply elastic or visco-elastic constants of composite materials. In the plastic region (or at breakage) they allow calculation of the characteristics of the boundary surface. The idea is to replace the real material by an equivalent, homogeneous material.

2.1 Boundary methods

Boundary methods have given a variety of formulae ranging from the simplest to the most complicated. They are derived from known theorems relating to elastic potential energy. Two approaches are used, one kinematically allowable, the other one statically allowable. If the geometry of the medium can be sacrificed, then the simplest formulae are those of Reuss and Voigt. However, when the mechanical characteristics of the two constituents (or phases) of the material are very different one from another (elastic constants and specific gravities) the boundaries are very elongated and divergent and of little usefulness.

2.1.1 The Hill theory

This theory[8], arising out of the work of Kroner[9] gives an improvement. It applies, particularly, to isotropic materials with spherical inclusions.

2.1.2 The Hashin and Rosen variational methods

These authors[10] have made an important progress in the field of assessment calculations, concerning materials having hexagonal or random distributions of unidirectional fibres. These fibres can be hollow or solid (Fig 2).

The representative elementary volume (R.E.V.) of the orthotropic material is defined. In the case of a hexagonal distribution, the admissible fields are those which produce either a constant deformation or a constant constraint in the space contained between the hexagon (R.E.V.) and the inscribed circle of the cross section. The problem is restored to a classical elasticity problem, in a cylinder of radius r_i with limiting conditions of the type:

$$u_i = \bar{\varepsilon}_{ij} x_j \quad \text{or} \quad F_i = \bar{\sigma}_{ij} n_j \tag{1}$$

Displacement on the boundary S — Surface density of forces

In this way, Hashin supplied the boundaries for the five elastic constants in the case of hexagonal symmetry and six in the case of random distribution.

2.1.3 Variational methods using polarisation tensors

Hashin and Shtrikman[11] have made a further improvement. Instead of minimising (or maximising) the potential energy of the admissible fields, they minimise (or maximise) a certain function U, defined on a functional space $E(V)$, where V is the volume and $E(V)$ is chosen as the product of the space of kinematically admissible fields and the space of polarisation constraints p_{ij}.

$$\sigma_{ij} = C^0_{ijk\ell} \varepsilon_{k\ell} + p_{ij} \tag{2}$$

where σ_{ij} and $\varepsilon_{k\ell}$ are the constraints and the deformations and the exponent 0 refers to the comparison medium.

The originality of the method lies in the fact that U is not defined uniquely by the material M being studied, but also by the reference material M_0, having the same geometry and being subjected to the same boundary conditions. The step is similar to that adopted in electricity for the study of dielectrics.

The statically admissible fields can also be used, together with a polarisation displacement, by prescribing a force field on S .

Alblas and Kuipers[12] have generalised the results of Hashin and Strikman in a space $E_1(v)$ containing $E(v)$ by modifying the function U .

Side by side with these variational formulae, and for industrial applications, there are many assessment methods, some semi-empirical (Chamis and Sendeckyj[13]) and others which take account of faults in the composite material (fibre contiguity, imperfect alignment, etc), such as that by Tsai[14].

2.1.4 Elastostatic methods

Chevalier[15], dealing with the hexagonal fibre network, used a representative, elementary volume, as did Hashin. He proposed the replacement of the hexagonal (R.E.V.) contour by a circular one which maintains the percentage of the fibres by volume. He worked with materials having transverse isotropy.

2.1.5 The elastodynamic method

This method is advocated by Behrens[16] as being useful for long waves applied to a variety of periodic, composite materials (lamellar composite materials, composite materials with anisotropic fibres or with sloping elastic constants, composite materials with peculiar symmetry, etc). Behrens uses plane waves in the periodic medium.

The methods given above are summarised, for comparison, in Table 1, which is taken from Chevalier[17]. Fig 3 and Table 2 give some indications on three assessment methods applied to glass fibre composite materials.

To conclude this section, it can be said that without prejudice of knowing the elastic constants of each phase of the material and the percentage of fibres by volume, there are now available, for materials with transverse isotropy, methods of assessment calculation which are making their mark in industrial applications. In my opinion, the finite elements, in spite of many attempts, have not produced any significant contributions in this field.

2.2 Assessment calculations in linear viscoelasticity

The use of the correspondence principle allows the viscoelastic constants of a material to be obtained from elasticity formulae. However, existing calculations are only true in the context of perfect adhesion between the fibres and the matrix and do not take into account the diffraction of waves at the interfaces. On this subject the work of Hashin[18] and Chevalier[19] should be noted.

2.3 Assessment calculations in the plastic state or at breakage

Kelly and Davis[20] have used them for unidirectional, composite materials. They examined the following three distinct processes:

(a) breakage controlled by the resistance to simple fracture along the fibre axes;

(b) transverse resistance in a direction perpendicular to (a);

(c) pure shear resistance of the matrix.

2.3.1 The MacLaughlin approach

Limit analysis has been used with more or less success by various researchers into multilayer composite materials. MacLaughlin and Batterman[21] have considered the extreme simplified cases (high or low percentages of fibres) which, unfortunately, do not fall within the normal percentage range (60% fibres). Majumdar and MacLaughlin[22] have attempted to state the general principles of the application of limit analysis methods in the search for a boundary surface. Their equations have some weaknesses.

2.3.2 The Le Nizerhy approach

In particular, the work of Le Nizerhy[23] should be mentioned. He has advocated a consistent method for enclosing the site of the mean constraint vector by means of a lower envelope (approach through constraints) and an upper envelope (dual approach). In the case of a perfectly-plastic rigid system, it requires static and kinematic approaches to the boundary surface (classical limit analysis). Taking into account the fragile, ductile rupture of the fibres, Le Nizerhy was led to consider the calculation, using only the hypothesis of convexity as criterion, and with the mathematical formalism unchanged.

The material layout takes into account the periodic distribution of fibres both in each layer and in the resin which forms the homogeneous layers. The loading parameters Q_i are introduced as being the mean constraints (with close homothetics) in a macroscopically homogeneous volume of the composite material. The deformation velocity vector $\dot{\underline{q}}$ associated with the vector Q in the expression for the principle of virtual powers, is expressed, this latter taking the form:

$$\int_{\Omega} tr(\underline{\underline{\sigma}} \cdot \underline{\underline{d}})d\Omega = \sum_{i=1}^{6} Q_i q_i = V(\Omega)\bar{\sigma}_{ij}\bar{d}_{ij} \tag{3}$$

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From the definitions of the mean quantities, $\bar{\sigma}_{ij}$ and $\bar{d}_{ij}$, can be obtained an interior envelope with potentially supportable loading surfaces, by trying to obtain the colvex envelope of all the calculated loadings Q_i and an exterior envelope, using a classical method of approach for the envelope of a family of planes. Fig 4 shows an outline of how the method can be used in an application affecting multilayer composite materials with boron fibres.

3 PROPAGATION OF WAVES AND VIBRATIONS IN COMPOSITE MATERIALS

It is in this field that a large number of theoretical studies are to be found. In order to grade them, the first thing is to compare the wavelengths λ with respect to the dimensions of the elementary network, d , of the composite material. If λ/d is large, the behaviour of the composite material is that of an equivalent, homogeneous material having the elastic characteristics given by the assessment calculations. In this context, there is available what is known as the theory of effective moduli. It is then possible to tackle the study of the vibration of rods, plates and shells, on the one hand, and the propagation of long waves in composite materials on the other. This theory finds its limitations very rapidly when λ/d approaches unity. It is then necessary to use more elaborate theories which could take better into account the dispersion of waves at higher frequencies. In this context, the theory of continuous media with micro-structure appears to be the most promising. Another approach is that of the so-called theory of mixtures which uses the interaction between the components of the composite material. Variational methods which affect periodic networks have been proposed by various authors and have given rise to recent interesting improvements by Nemat-Nasser[24]. The theory of continuous media based on asymptotic developments should also be noted.

3.1 Theory of effective moduli

3.1.1 Wave propagation

The mean constraints $\bar{\sigma}_{ij}$ and the mean deformations $\bar{\varepsilon}_{ij}$ in a representative elementary volume are defined and the elastic constants of the equivalent, anisotropic, homogeneous medium, $C^{*}_{ijk\ell}$ are calculated

$$\bar{\sigma}_{ij} = C^{*}_{ijk\ell}\varepsilon_{k\ell} \quad (4)$$

In an indefinite medium, the study of wave propagation using equation (4) does not allow the dispersion of the waves to be taken into account. However, this theory does provide a rapid, mechanical characterisation of the medium by ultrasonic methods using long waves.

3.1.2 Vibrations of rods and plates

A certain number of classical vibration studies for isotropic rods and plates can be transposed to composite materials. For rods, these include the work of McNiven[25], Le Nizerhy-Vinh-Chevalier[26], Vinh-Nugues[27] and Touratier[28] relating to approximate theories on rods.

It is a matter of extending the known approximate theories of Hermann-Mindlin, of Volterra (longitudinal vibrations), of Timoshenko's method (flexion vibrations) and of Barr-Engstrom's[29] method (torsional vibrations).

It should be noted, in passing, that in general the dispersion of waves for the three modes of decoupled vibrations is much more pronounced for composite materials than for isotropic ones.

This dispersion is due to the strongly anisotropic character of the medium studied, which is shown, particularly, in the high ratio between the longitudinal Young's modulus, E_1 , and the shear modulus μ :

$E_1/\mu \approx$ 20 to 50, instead of about 3, for isotropic materials.

Here and now, for the choice of displacement fields, polynomials of a high degree must be used to translate, correctly, the dispersion of phase and group velocities. Applying dynamic torsion to beams of rectangular cross section[27], the Saint-Venant theory, which relates to the buckling function, reaches its limits in the study of the higher modes.

Rods (whose axes are not confounded with axes of symmetry of the material) give rise to coupled waves. Several approximate theories exist, such as those by Abarcar and Cunniff[30] and Le Nizerhy[31]. They all require complementary studies. The method advocated by Mindlin[32] of the double series for the displacement components should be noted.

$$u_i = \sum \sum x_2^m x_3^n u_i^{m,n}(x_1, t) = \sum x_2^m x_3^n u_i^{m,n} \qquad (5)$$

where x_1 is the beam axis and t is time. The number of terms taken in equation (5) depends upon the modes of vibration studied. For plates, the well-known approximate theory by Mindlin has been extended to composite materials by Whitney[33].

Amongst the work in France should be noted that of Verchery[70] on composite plates in a static state. This work is capable of being extended to the dynamic state.

3. Theories of continuous media with discrete structures

A number of theories with microstructures have been proposed, amongst which are the works of Mindlin and Tiersten[34], the theories of Toupin[35] and Truesdell[36] and the so-called micropolar theory of Eringen[37]. This latter tends to generalise the other theories. A comparative study by Eringen[38] brings out the differences between these theories (restrictions imposed on the spatial microrotation vector $\phi_k = \frac{1}{2} e_{k\ell m} u_{m,\ell}$ where u_m is the displacement vector; the presence of the couple ℓ_k per unit mass and of the acceleration u_k in the behaviour laws; omission of the micropolar rotatory inertia term; final dependence of the constraint vis-à-vis the applied loads and the inertia, etc).

Most of the theories on microstructures are concerned with linear, isotropic materials.

As regards the propagation of waves in an indefinite medium, the following theoretical results by Eringen should be noted.

For plane waves, Fig 5 shows that in the direction of propagation, n , there is a microrotational plane wave in addition to the classical longitudinal wave. For the microrotational wave, propagation is only possible if the frequency exceeds a certain cut-off frequency, Fig 6.

For transverse waves, in addition to the classical transverse displacement wave, U , Eringen has found a wave with microrotation ϕ which is orthogonal to both the direction of propagation, n , and the displacement U . This latter transverse wave with microrotation only exists for frequencies greater than a critical frequency. Below this, it degenerates into an evanescent wave attenuated by distance. For surface waves, Eringen and Suhubi[38] have predicted a second type of dispersive wave. Here, too, no experimental confirmation has yet been put forward. Amongst the theories for discrete structures applicable to composite materials should be mentioned two proposals, one by Hermann, Sun and Achenbach[39], known as the effective stiffness theory and the other, the Drumheller-Bedford[41] theory which uses a second order approximation.

3.2.1 Theory of effective stiffness

This was developed, originally, to interpret dynamic effects such as the dispersion of waves in stratified media (multilayers). Fig 7 represents a rectangular disposition of unidirectional fibres in a plane cross section. In a grid (k, ℓ) it is a matter of adopting series for displacements. The following developments can be adopted fairly well in the fibre (f) and matrix (m):

$$u_i^{f(k,\ell)} = \bar{u}_i^{(k,\ell)} + r \cos\theta \,.\, \psi_{2i}^{f(k,\ell)} + r \sin\theta \,.\, \psi_{3i}^{f(k,\ell)} \qquad \text{(fibre)}$$

$$u_i^{m(k,\ell)} = \bar{u}_i^{(k,\ell)} + a \cos\theta \,.\, \psi_{2i}^{f(k,\ell)} + a \sin\theta \,.\, \psi_{3i}^{m(k,\ell)}$$

$$+ (r-a)\cos\theta \,.\, \psi_{2i}^{m(k,\ell)} + (r-a)\sin\theta \,.\, \psi_{3i}^{m(k,\ell)} \qquad (6)$$

with $r > a$ matrix .

The physical interpretation is simple. $\bar{u}_i^{f(k,\ell)}$ represent overall displacements and $\psi_{ni}^{m(k,\ell)}$ $(n = 2, 3)$ represent local displacements. The two types of displacements are defined for discrete values of x_2 and x_3 and are continuous functions of the variable x_1 and time. Since it is not possible to satisfy, exactly, the conditions for continuity of the displacements at the interfaces between neighbouring cells, Sun and Achenbach suggest a continuity for the mean displacements.

On the basis of these series developments (which use Legendre polynomials) they have evaluated the kinetic and potential energies whose means are defined at the fibre centres. The transition from a discrete lattice to a continuous, homogeneous model is accomplished by defining the fields for the kinematic and dynamic variables which are continuous for all the coordinates. Thus the energy deformation density contains not only the effective moduli terms, shown previously, but also terms with constants arising from combinations of the geometric and elastic properties of each phase. The equations are obtained by the Hamilton principle used in conjunction with the continuity relationships and Lagrange multipliers.

As a particular case, the researchers have studied multilayer plates where each layer is isotropic. Fig 8 shows the effective stiffness theory with a first order approximation. It shows that for the phase velocity agreement is good for very low modes (number of wave $k < 2$ with respect to the exact theory of Rytov[40]).

The model with a first order approximation translates, correctly, the dispersion of shear modes but not the other modes. A second order approximation, Fig 9a, gives a better approximation to the exact theory.

3.2.2 General theory of effective stiffness

As an application of the previous theories, the problem of a semi-indefinite or finite body subjected to a permanent or transitory displacement and/or the limiting constraint conditions, is of great importance. These problems have been formulated by Bedford and Drumheller[41]. They can be applied to the highest order of effective stiffness theories. These two authors introduce an external, plane surface and systems of coordinates x_1, x_2 (Fig 9b) relative to a multilayer composite material, as well as local coordinates $\left(x_1^f, x_2^f\right)$ and $(\bar{n}, \bar{s})$ whose origins are situated on the fibre axis. The displacement vector, u_i^{fk}, in the kth layer can then be written as:

$$u_i^{fk} = \hat{u}_i^{fh}\left(\bar{u}_i^f, \bar{x}_2^f, t\right) = \hat{\hat{u}}_i^{fh}(\bar{n}, \bar{s}, t) \quad . \tag{7}$$

The limiting conditions for the displacements in terms of the distribution of *specified displacement* $U_i^{fk}(\bar{s}, t)$ *at the external* boundary of the layer k are:

$$\hat{u}_i^{fk}\left(\bar{x}_i^f, \bar{x}_2^f, t\right)_{\bar{n}=0} = \hat{\hat{u}}_i^{fk}(0, \bar{s}, t) = U_i^{fk}(\bar{s}, t) \quad . \tag{8}$$

Then $\hat{u}_i^{fk}$ and U_i^{fk} are developed in terms of $\bar{s}$ and the development is of the same order as the development of displacement vectors in the theory adopted.

By equalising terms in the same power of $\bar{s}$, a set of limiting conditions is obtained:

$$\left.\begin{aligned} u_i^{fk}(0,0,t) &= U_{0i}^{fk}(t) \\ \frac{\partial \overset{*}{u}_i^{fk}}{\partial \bar{s}}(0,0,t) &= V_i^{fk}(t) \\ \frac{\partial \hat{\hat{u}}_i^{fk}}{\partial^2 \bar{s}^2}(0,0,t) &= W_i^{fk}(t) \end{aligned}\right\} \quad . \tag{9}$$

This procedure is direct and shows the relationship between the limiting conditions and the distributions of displacements and constraints over the external surface.

The problem of the accuracy of the effective stiffness theory, with its limiting conditions, has been discussed by Bedford and Drumheller[41]. They have shown that the microstructure theory, with a finite number of degrees of freedom, has only limited possibilities for modelling the distributions of displacements and constraints at the boundary.

3.3 <u>So-called theory of mixtures, or of media with interactions</u>

This is a matter of a theory used, originally, in geology and recently adapted to composite materials by Lemprière[42], Bedford and Stern[43] and Hegemier[44].

It is assumed that each phase of the composite material is subjected to individual deformations. The microstructure of the composite material is shown by the interactions between the constituents. Some indications of how this method is applied to multilayers will be given here.

By integrating the equations of movement in the deformation plane we have:

$$\sigma^{(\alpha)}_{ij.j} = \rho u^{(\alpha)}_{i,\ell t} \tag{10}$$

where α indicates one of the two phases of the medium, with respect to the variable $x_3^{(\alpha)}$ in the direction of the thicknesses $h^{(\alpha)}$. Equations are obtained of the form:

$$h^{(\alpha)} \frac{\partial \sigma^{(\alpha)}_{11}{}^{e}}{\partial x_1} - h^{(\alpha)}\rho^{(\alpha)} \frac{\partial^2 u_1^{(\alpha)}{}^{e}}{\partial t^2} = -\tau^{(\alpha)}_{13}\left(x_1, h^{(\alpha)} t\right) \tag{11}$$

where the exponent e indicates a mean.

$$\left(\; \right)^{(\alpha)e} = \frac{1}{h^{(\alpha)}} \int_0^{h^{(\alpha)}} \left(\; \right)^{(\alpha)} dx_3^{(\alpha)} \tag{12}$$

and assures the continuity of the term $\tau^{(\alpha)}_{13}$, which is asymmetrical with respect to $x_3^{(\alpha)}$:

$$-\overset{(1)}{\tau}_{13}(x_1,\overset{(1)}{h},t) \;=\; \overset{(2)}{\tau}_{13}(x,\overset{(2)}{h},t) \;=\; \tau^{*}_{13}(x_1,t) \quad . \tag{13}$$

Equations of movement are obtained, of the form:

$$\frac{\partial \overset{(1)}{\sigma}{}^{p}_{11}}{\partial x_1} - \overset{(1)}{\rho}{}^{p}\,\frac{\partial^2 \overset{(1)}{u}{}^{p}_{1}}{\partial t^2} \;=\; \rho_c \beta \tag{14}$$

where the exponent p indicates partial constraints or displacements.

$$\overset{(\alpha)}{\sigma}{}^{p}_{11} \;=\; \overset{(\alpha)}{V}\,\overset{(\alpha)}{\sigma}{}^{c}_{11} \qquad\qquad \overset{(\alpha)}{\rho}{}^{p} \;=\; \overset{(\alpha)}{V}\;\overset{(\alpha)}{\rho} \tag{15}$$

$$\overset{(\alpha)}{V} \;=\; \overset{(\alpha)}{h}/(\overset{(1)}{h} + \overset{(2)}{h}) \quad ,$$

where $\overset{(\alpha)}{V}$ is the volume fraction,

$$\rho_c \beta \;=\; \tau^{*}_{13}/(\overset{(1)}{h} + \overset{(2)}{h}) \quad , \tag{16}$$

with

$$\overset{(\alpha)}{u}{}^{p}_{1} \;=\; \overset{(\alpha)}{u}{}^{c}_{1} \quad . \tag{17}$$

Thus the $\rho_c\beta$ term in equation (14) represents the transfer of the amount of movement from one phase to another. Hence it corresponds to a volume force equivalent to an interaction due to shear at the interfaces.

The basic problem is the determination of these interaction terms $\rho_c\beta$ and the equations of behaviour for each phase of the composite material. For this, Hegemier *et al*[44] have suggested an asymptotic development as a function of x_3 for the constraints and displacements.

The equations of behaviour are of the form:

$$\left.\begin{aligned}\overset{(1)}{\sigma^{p}_{11}} &= C_{11}\left(\frac{\partial \overset{(1)}{u^{e}}}{\partial x_1}\right) + C_{12}\left(\frac{\partial \overset{(2)}{u^{e}}}{\partial x_1}\right)\\ \overset{(2)}{\sigma_{11}} &= C_{12}\left(\frac{\partial \overset{(1)}{u^{e}}}{\partial x_1}\right) + C_{22}\left(\frac{\partial \overset{(2)}{u^{e}_1}}{\partial x_1}\right)\end{aligned}\right\} \quad (18)$$

which express the constraints in terms of the composite material constituents and the displacement gradients in each phase.

Figs 10 and 11 show the phase velocities in terms of the frequency for two types of multilayer composite materials. The agreement between theory and practice is good for the low frequencies.

3.4 Theory of continuous media with microstructure based on asymptotic developments

Several formulae exist for the propagation of waves, such as those of Hegemier and of Nayfeh[47].

The method given by Nayfeh is of particular interest when the number of layers exceeds three in each cell and the pattern is repeated particularly in one direction. For lack of space, only the steps adopted for a unidirectional problem will be given here.

(a) As in the theory of mixtures, the equation of movement is integrated with respect to the variable x in each elementary cell $(0 < a < m)$.

(b) By assuring the conditions of continuity between one cell and another, the relationships are sought which give the constraints $\sigma_{\alpha}^{(k)}(0)$ and $u_{\alpha}^{(k)}(0)$, the displacements at the median plane of each layer.

(c) Next, the constraints and displacements found above are developed as series, as functions of the local coordinate $y_{\alpha}^{(k)} = x - x_{\alpha}^{(k)}$, where x is the overall coordinate and $x_{\alpha}^{(k)}$ is the local coordinate in the α layer of the kth cell. Thus a range of finite difference equations is obtained for the spatial and differential coordinates with time.

(d) Then a small parameter $\varepsilon = \Delta/\ell$ is introduced, where Δ is the total thickness and ℓ is a macrodimension of the problem, which could, for example, represent the wave length λ. When $\varepsilon \to 0$ the phase velocity obtained is that given by the theory of homogenised, non-dispersive media.

Amongst the approximate theories put forward up to now with models of wave dispersion for the lowest modes, the method of asymptotic development gives a greater accuracy than do the others.

3.5 Variational methods

Kohn[3] has applied the theory of Floquet, Bloch and Brillouin to composite materials. Lee[48] has recently reviewed various variational methods in detail. The variational principles are developed in an integral form in a cell which represents the representative elementary volume. The deformation energy and the complementary energy have been used. In passing from the fibre to the matrix, the discontinuities in certain constraints and deformations at the interfaces are taken into account. A Rayleigh-Ritz procedure has been applied to calculate the dispersion relationships for any propagation direction whatsoever. So far as is known, the studies by Lee have been confined to multilayers (periodicity in one direction). The credit for a generalisation of such methods and for an application to several cases rests with Nemat-Nasser[24,49] who has, in addition, proposed a dual Rayleigh coefficient (based on the constraints). For various situations (pronounced density discontinuity, discontinuity of the elastic constants) these variations of the coefficients show the Rayleigh coefficient to be adequate. A new Rayleigh coefficient, based on mixed fields, appears to be very effective and gives the upper boundaries for particular values with needing recourse to the usual orthogonalisation of the test functions.

3.5.1 The Rayleigh displacement coefficient

This coefficient is:

$$\left.\begin{aligned} \lambda_R &= \frac{\langle \eta u', u' \rangle}{\langle \rho u, u \rangle} \\ \langle \rho u, u \rangle &= \int_{-\frac{1}{2}}^{\frac{1}{2}} \rho u \, . \, v^* dx \end{aligned}\right\} \qquad (19)$$

where ρ is the density,

u is the displacement in reduced variable,

η is the elastic coefficient,

$*$ indicates the conjugated complex quantity,

$'$ indicates differentiation with respect to x .

The periodicity of the medium is expressed as:

$$u(\tfrac{1}{2}) = u(-\tfrac{1}{2})e^{iQ} \tag{20}$$

where Q is the wave number.

If L is positive, auto-associated operators, for harmonic waves we have:

$$Lu = -\frac{1}{\rho(x)}\,[\eta(x)u']' = \lambda u \quad . \qquad -\tfrac{1}{2} < x < \tfrac{1}{2} \tag{21}$$

At the points x_j $(j = 1,2)$, of discontinuity η, we should have the following conditions:

$$\begin{aligned} \eta(x_j^-)u'(x_j^-) &= \eta(x_j^+)u'(x_j^+) && j = 1 \text{ or } 2 \\ x_j^{\pm} &= \lim_{\varepsilon\to 0}(x_j \pm \varepsilon) && \varepsilon \to 0 \end{aligned} \tag{22}$$

Equations (20) and (21) can be satisfied by taking for the displacement:

$$\bar{u} = \sum_{\alpha=0}^{\pm M'} U_\alpha \exp i(Q + 2\pi\alpha)x \quad . \tag{23}$$

The method of calculation is the classical one:

$$\frac{\partial\lambda(\bar{u})}{\partial U_\alpha^*} = 0 \qquad \alpha = 0,\pm 1,\ldots,\pm M' \quad . \tag{24}$$

When η has discontinuities or varies very rapidly, this method is ineffective.

3.5.2 The dual Rayleigh coefficient

Assuming that:

$$D = \frac{1}{\eta}, \qquad R = \frac{1}{\rho} \qquad \text{and} \qquad D\sigma = u' \tag{25}$$

we have

$$(R\sigma')' + \lambda D\sigma = 0 \quad , \qquad -\tfrac{1}{2} < x < \tfrac{1}{2} \tag{26}$$

$$\sigma(\tfrac{1}{2}) = \sigma(-\tfrac{1}{2})e^{iQ} , \qquad (R\sigma')_{x=\frac{1}{2}} = (R\sigma')_{x=-\frac{1}{2}} e^{iQ} . \tag{27}$$

The dual coefficient is:

$$\bar{\lambda}_R = \frac{\langle R\sigma', \sigma'\rangle}{\langle D\sigma,\sigma\rangle} , \qquad \bar{\sigma} = \sum_{\alpha=0}^{\pm M'} S_\alpha \exp i(Q + 2\pi\alpha)x . \tag{28}$$

$\bar{\lambda}_R$ is made stationary:

$$\frac{\partial\bar{\lambda}_R(\bar{\sigma})}{\partial S^*_\alpha} = 0 , \qquad \alpha = 0,\pm 1,\ldots,\pm M' .$$

When $R = 1/\rho$ is not continuous, $\bar{\lambda}_R$ is not effective.

3.5.3 <u>A new coefficient</u>

Taking the mixed fields σ and u we obtain:

$$\lambda_N = \frac{\langle \sigma,u'\rangle + \langle u',\sigma\rangle + \langle D\sigma,\sigma\rangle}{\langle \rho u,v\rangle} . \tag{29}$$

It can now be shown that:

$$\lambda_R(u) \geqslant \lambda_N(u,\sigma) \tag{30}$$

and that

$$\lambda_R^{(p)} = \lambda_N^{(p)} > \lambda_p , \quad \text{if } \eta \text{ is a constant} .$$

3.5.4 <u>A new dual coefficient</u>

$$\bar{\lambda}_N = \frac{\langle W,\sigma'\rangle + \langle \sigma',W\rangle - \langle \rho W,W\rangle}{\langle D\sigma,\sigma\rangle} \tag{31}$$

with

$$\bar{\lambda}_R(\sigma) > \bar{\lambda}_N(\sigma,W)$$

and if ρ is a constant:

$$\bar{\lambda}_R(\sigma) = \bar{\lambda}_N(\sigma,W) > \lambda_p .$$

3.5.5 Improvements in the new coefficients

This has led Nemat-Nasser to propose a modification to the test functions $\bar{u}$ and $\bar{\sigma}$, as in equation (22), so that the following continuity condition is not violated.

$$R(x_j^-) \cdot \sigma(x_j^-) = R(x_j^+) \cdot \sigma'(x_j^+) \qquad \text{with } j = 1 \text{ or } 2 . \quad (32)$$

Moreover, he ensures that the new test functions $\hat{u}$ and $\hat{\hat{\sigma}}$ are such that $\eta\hat{u}'$ and $R\hat{\hat{\sigma}}'$ are continuously differentiable in $[-\frac{1}{2},\frac{1}{2}]$.

For the unidimensional problem the modified functions are:

$$\hat{u} = \sum_{\alpha=0}^{\pm M'} S_\alpha \left\{ (\exp iQ - 1)^{-1} \int_{-\frac{1}{2}}^{\frac{1}{2}} D(\xi) \exp i(Q + 2\pi\alpha)\xi d\xi + \int_{-\frac{1}{2}}^{x} D(\xi) \exp i(Q + 2\pi\alpha)\xi d\xi \right\} \quad \ldots\ldots(33)$$

$$\hat{\hat{\sigma}} = \sum_{\alpha=0}^{\pm M'} W_\alpha \left\{ (\exp iQ - 1)^{-1} \int_{-\frac{1}{2}}^{\frac{1}{2}} \rho(\xi) \exp i(Q + 2\pi\alpha)\xi d\xi + \int_{-\frac{1}{2}}^{x} \rho(\xi) \exp i(Q + 2\pi\alpha)\xi d\xi \right\} \quad \ldots\ldots(34)$$

and a new lower boundary, $\hat{\lambda}_R^P$, and a new upper boundary, $\hat{\hat{\lambda}}_N$, are obtained at the price of an improvement in u and σ.

Nemat-Nasser, Fu and Minagawa[50] have also provided boundaries for the natural frequencies in three-dimensional problems. These researchers have made numerous applications of the method, which indicates the interest shown in it.

Mention should be made of the work of Gotteland[51] who has carried out a systematic study of the various variational formulae with a view to the applications to multilayer plates by finite elements.

3.6 Various models which take wave characteristics into account

The propagation of waves in composite materials is always accompanied by a attenuation. This can be explained by:

(a) the periodic character of the lattice;

(b) the viscoelastic properties of the constituents;

(c) the diffraction and diffusion of waves by the fibres, by interfacial defects, by holes in the matrix, by bad fibre alignment, etc.

Taken as a whole, the problem is, theoretically, of great complexity. In practice, there have been only tentative, partial explanations of certain aspects relating to wave attenuation.

3.6.1 Modelling by analogy with a continuous medium

This has been suggested by Drumheller and Sutherland[52], using the results obtained by Brillouin[1] for discrete lattices based on the Floquet-Bioch theorem relating to the propagation of a harmonic disturbance. The wave has a periodic dispersion spectrum. The frequency curve, in terms of the wave number $R = 2\pi/\Lambda$, where Λ is the wavelength, has frequency bands in which no wave can be propagated and for a given value of f there is an infinite number of values of k.

The classical dispersion relationship can be used for the lattices, giving:

$$\cos(kh) = \frac{(Z+1)^2}{4Z}\cos(\omega t^+) - \frac{(Z-1)^2}{4Z}\cos(\omega t^+ t_r) \qquad (35)$$

where Z is the ratio of the acoustic impedances of the two layers,
t^+ is the sum of transit times in the elementary cell,
t_r is the ratio of the transit time difference to the sum of transit time,
h is the total thickness.

By analogy with a continuous medium, the properties of this latter can be used, and are as follows:

(i) frequencies filtered in the by-pass filter;

(ii) at almost zero frequencies, the lattice and the composite material have the same longitudinal phase velocity. Then equation (35) is developed and gives the relationship for the dispersion of waves from the lowest mode;

(iii) a redistribution of masses is undertaken in the lattice to adapt the dispersive characteristics of the ideal, periodic lattice to the composite material.

This method, which is very simple to apply, can be used to study shock waves.

3.6.2 The Sve method

The Fourier integral has been used to simulate the remote field of constraints near to shock wave fronts by assuming that the predominant modes are those with the lowest frequencies. Dispersion is included by adopting a complex wave number. Sve studies the differences and the common points between the geometric dispersion and the spatial attenuation of the waves. He uses the Airy function.

3.6.3 Interpretation of dispersive effects by the Boltzmann relationship

Christensen[54] has suggested the use of dielectric theory as a possible approach, since the two basic mechanisms of dielectrics have a certain analogy with those of composite materials:

(i) there is the loss due to displacement of loads which are connected, elastically, to an equilibrium position;

(ii) there is the loss due to the transition of loads or dipoles between the equilibrium positions, separated by a potential barrier.

According to Christensen, case (i) can give a direct characteristation of elastic media with periodic layers and case (ii) has been transposed to a homogeneous, viscoelastic medium.

In this way, Christensen has used this analogy to re-examine the dispersion relationship obtained by Kohn in the form:

$$\omega^2 = C_0^2 k^2 - \beta k^4 \qquad (36)$$

where C_0 and β are parameters which express the layers and their geometries in terms of the elastic characteristics.

He has also examined the random medium by adopting a relaxation function of the form:

$$G(t) = \tilde{G}e^{-t/\tau} + G_0$$

where $\tilde{G}$, τ and G are unknown parameters which are to be determined. He obtains the attenuation in the form:

$$\eta = \frac{\omega^2 h\left(\frac{h_1}{h_2}\right)^2\left(\frac{h_2}{h}\right) C_0 C_{max}(\rho_1 G_1 - \rho_2 G_2)^2}{4G_1^2 G_2^2} \qquad (37)$$

where C_{max} is the maximum wave velocity,
ρ_1, ρ_2 are densities,
G_1, G_2 are the moduli of elasticity of the layers,
h_1, h_2 are the thickness,
ω is the pulse rate.

Equation (37) allows the order of magnitudes of the attenuations of waves in magnesium and aluminium to be remeasured.

3.6.4 Diffraction and diffusion of elastic waves

As far as I am aware, very little study has been given to this problem, when its complexity is taken into account. It is a matter of studying the diffusion of waves due to the frequency of fibres sunk in a matrix. Bose and Mal[55] have examined the case of the transverse wave being propagated at right angles to the fibres and having the parallel polarisation of the fibres. It is a case of random fibre distribution.

These authors have been able to evaluate the complex wave number of the composite material. The imaginary part of the wave number is related to the geometry of the fibre lattice. They have been able to calculate the specific absorption in terms of the fibre concentration.

4 SOME EXPERIMENTAL STUDIES

4.1 Measurements of elastic constants in micropolar elasticity

4.1.1 Static measurements

It has been shown, earlier, that certain microstructure theories have been used, successfully, in the study of wave dispersion.

A serious problem arises, namely how, within the scope of these theories, can the elastic constants of materials be evaluated in sufficient numbers? Experimentally, this question underlines some difficulties. In research into the laws of behaviour in micropolar elasticity, certain attempts have been made which merit further notice.

Thus Gauthier and Jashman[56] have made measurements on an artificial, micropolar medium formed from aluminium particles in an epoxy resin matrix. According to Eringen, the behavioural laws are of the form:

$$\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + (\mu + \chi)\varepsilon_{ij} + \mu\varepsilon_{ij} \qquad (38)$$

$$m_{ij} = \alpha\phi_{k,k}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i} \qquad (39)$$

where σ_{ij} is the constraint,
ε_{ij} is the deformation,
ϕ_i is the microrotation vector,

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m_{ij} is the restraining couple,

λ and μ are the classical Lamé moduli, and

χ, α, β and γ are the form constants inherent in the micropolar theory to be determined.

For an isotropic body six elastic constants have to be evaluated. From equations (38) and (39), χ has the dimensions of pressure whilst α, β and γ have the dimensions of forces (couple/length).

The measurements can use well-known methods of classical elasticity (traction, torsion, shearing, circular flexion, etc) for rods and plates. The analysis formulae are more complex, for example in the case of traction which gives Young's modulus and the Poisson number in the following forms:

$$\left.\begin{aligned} E &= (2\mu + \chi)(3\lambda + 2\mu + \chi)(2\lambda + 2\mu + \chi)^{-1} \\ \nu &= \frac{\lambda}{(2\lambda + 2\mu + \chi)} \end{aligned}\right\} \qquad (40)$$

For torsion and flexion it has been possible to demonstrate the characteristic lengths which allow a definition of the limits within which micropolar elasticity measurements hold good.

Gauthier has shown that in resolving three problems at the limiting conditions he has been able to evaluate the six constants.

For shear measurement, use can be made of holographic interferometry. For torsion, the displacements can be measured by double exposure hologram and a strongly-coherent laser beam. These optical methods avoid physical contact and hence local disturbances in the areas of interest.

4.1.2 Dynamic measurements

Certain methods of physical measurement can be used to evaluate the elastic constants. A list of these methods is shown in Table 3.

In general, the optical methods given are concerned with short wavelengths and can be used when the characteristic micropolar wavelengths are of the same order of magnitude as the dimensions of the lattice ($\simeq 10^{-7}$ cm). It should be noted that Raman spectroscopy is reserved for crystals.

As Askar[57] has so rightly noted, there is a large quantity of experimental results on crystals which await interpretation by theoriticians and experimental workers in the field of continuous media.

Amongst the experimental methods which can show up certain 'micropolar effects' (Table 2), ultrasonics and hypersonics are very serious candidates.

4.2 Mechanical characteristics of anisotropic composite materials

The techniques for measuring the elastic constants of non-polar elastic or viscoelastic bodies tend to increase. Particularly in the aeronautical field there are important developments in static or dynamic experimental techniques. Rods, plates and tubes have been used to this end. It should be mentioned that the measurement of certain 'non-diagonal' elastic constants (that is to say which are not situated on the diagonal of the corresponding matrix) avoids couplings (flexion-torsion, traction-torsion, etc) in quasi-static tests. If ultrasonics are used, it will be possible to use coupled waves whose use is more delicate than decoupled waves, as Curutchary[58], Vinh and Garceau[59] and Markham[60] have shown.

Fig 12 shows an elasticimeter with alternate flexion and torsion, which allows Young's moduli (E_i), shear moduli (μ_{ij}) and the coupling constants of composite materials to be determined on samples of rectangular (or circular) cross section. Taking into account the influence (which is not negligible) of the rotational inertia effect (in flexion) and the shearing effect, the equations of motion are of the Timoshenko type with, possibly, coupling terms[26]. In alternate torsion, for rectangular cross sections, warping[27] must be taken into account.

In other respects, the use of samples of finite length complicates research into the natural modes, even by small amounts.

Fig 13 shows some experimental equipment which uses ultrasonics[58]. Fig 14 shows the slowness (inverses of phase velocities) diagrams relating to a unidirectional Kevlar (fibre in aromatic polyamide) composite material. It can be seen that for an angle of incidence of more than 13°, the longitudinal wave is no longer transmitted. The composite material therefore plays the role of a very directional spatial filter. The use of coupled waves for the evaluation of the last 'non-diagonal' constant gives rise to Fig 15. The error perpetrated on the value of the constant C_{1313} can have a strong influence on the value of C_{1133}.

4.3 Study of wave dispersion by ultrasonics

Propagation of expanding ultrasonic waves perpendicular to the layers has shown both waves of low mode (acoustic branch) and those of a higher mode (so-called optical branch). Fig 16 shows that the composite material behaves like a cut-off filter in certain frequency intervals. The effect of this property is to modify the frequency spectrum of the progressive wave, as is shown in Fig 17 (from Robinson and Lippelmeier[61]).

The wave dispersion can be shown, easily, by using several transducers having a range of natural frequencies.

4.4 Degeneration of ultrasonic waves

A decoupled wave, applied to a composite material, and being propagated along a direction of symmetry of the material, can be transformed into coupled waves (transversal and longitudinal waves). This happens if the fibres undergo directional variations. Hence this wave transformation can be used to study certain faults in fibre alignment (Weistman[62]).

4.5 Geometric and viscoelastic dispersion

Even in an indefinite medium formed from unidirectional fibres, the geometrical dispersion of the waves can be explained by fibre size (which behave as wave guides) and by their periodicity. The viscoelastic type of dispersion is due to the matrix (often of resin). These two types of dispersion exist together and can either act in the same sense or can neutralise each other, depending upon the modes[58,59].

4.6 Elastic waves in limited media

The study of surface waves[63], of plate waves, of Love waves (surface waves with transverse polarisation which are propagated between a substrate and a layer) and of Stoneley waves (interface waves) has not given rise, as far as I am aware, to theoretical and experimental studies on composite materials. The studies would certainly be more refined than those done on homogeneous, isotropic media but they could lead to very interesting applications, particularly in the study of adhesion between the layers of composite materials.

4.7 Evidence of special waves related to the micromorphic nature of the medium

Section 3.3 drew attention to particular waves with microrotation (longitudinal or transverse waves). As far as I am aware, these waves have not yet been demonstrated in the ultrasonic region. Special transducers will have to be designed to show this effect.

4.8 Shock waves

These are the ones most used for composite materials. Various methods have been adopted, such as those of Whittier and Peck[64], Sve and Okubo[65], Tauchert and Moon[66]. The same techniques are used as for conventional solids. Amongst others, these include methods using a mechanical projectile, explosive charges, shock waves in a gas and waves in a shock tube. The shock wave is propagated in the composite material and detection of the wave is done on the opposite face by a capacitative displacement pick-off, Fig 18. Wire resistance strain gauges can be embedded in the composite material to record the waves at various points.

Large amplitude transitory waves can be generated by the 'flying plate projectile' technique. This plate is made from a homogeneous material, mechanical and polished to form the head of the projectile, which is launched by a gas gun. The composite, in the form of a thick block, is attached to two adaptors ('buffers') which serve as connectors and which have been aligned, previously, with respect to the flying plate. Displacement measurements can be by the short-circuit contact method.

Piezo-electric crystals can be used or a laser interferometer (Michelson, with Doppler effect) which measures the displacement through a window[67].

Amongst the theories given earlier, it is the theory of media with interaction which is seen to be the most successful in the study of shock waves (equation (18)). The slower progress with the other theories is explained by the fact that these latter contain parameters from undetermined behavioural laws. The evaluation of these constants (or functions) by analytical or experimental methods is very critical, although certain attempts (such as those mentioned in section 4.1) are promising. Fig 19 shows the good agreement between theory and experiment in a shock test on carbon-phenolic resin composite materials.

4.9 Mechanical tests on bones

Bones are really biological composite materials. Experimental and theoretical studies devoted to composite materials can be transposed to the biomechanics of solids.

The anisotropy of bones is very pronounced and, moreover, is often complicated by a range of elastic constants. Guynemer[68] has made measurements to give the mechanical characteristics of bones.

Within this field, subsequent special studies should take into account the specific characteristics of bones. Rayleigh waves in cylinders could be used to study cortical layers.

5 CONCLUSIONS

Taking into account the fractional aspects of the composite materials considered here, it is difficult to draw any firm conclusions. At best, it can be said that in theory numerous formulae can be found for the study of waves and vibrations. The most simple versions of the theory of elasticity in microstructures, known as the effective stiffness theory, are capable of being used with success in multilayer composite materials. Variational methods are very promising. The same is true of asymptotic methods and the theory of media with interaction.

To my mind, the variety of theories available could cover all the experimental requirements without the necessity for further development. Any lack is with the experimenters, because the absence of exact solutions in numerous problems requires experimentation to decide between the approximate theoretical solutions proposed. It is likely that more and more they will be forced to use theories with microstructures. The use of multiple scales, one over all and the other for the elementary lattice, is an approach which compels recognition, more and more, to show up certain fracture or delamination phenomena which are of a microstructural order. To this end, attempts to remain within the framework of the classical theory of continuous media are already reaching their limitations. A more careful study of the field of constraints by means of a microstructure theory, of higher order than the existing ones, should assist experimenters to give a better explanation of the phenomena which arise at the matrix-fibre interface level.

Various physical methods (in particular optical methods and those using very high frequency ultrasonics) have not been used for composite materials. These new methods, as yet little used by engineers, could be capable of explaining certain acoustic phenomena already predicted, theoretically, by Eringen[37].

Hence composite materials offer a wide field of choice for interdisciplinary work between physicists and engineers.

Finally, at the calculation level, the question is to know whether or not the various existing theories can produce numerical solutions which are more effective than those obtained by direct examination of material in finite elements.

Table I

SUMMARY OF THE ASSESSMENT CALCULATION METHODS, TAKEN FROM CHEVALIER

Nature of the method	Author	Nature of the composite material	Advantages	Disadvantages
Variational (limits for modules)	Hill	Isotropic. Slightly different characteristics	Limits independent of material geometry. Documentary results. Any interfacial bonds. Pedagogical interest	Limits too wide if the phases are different
	Hashin-Rosen	Transverse isotropic	Use of material geometry. Documentary results	Relatively complicated calculations
	Hashin-Shtrikman	Isotropic (and even multiphase)	Limits closer to those given by Hill	Demonstration difficult, needing calculus of variances
Elasto-static	Pickett, Leissa and Clausen	Tetragonal orthotropic or transverse isotropic	Applicable to a large number of materials. Rigorous method	Needs a large computer. Numerical results. Long calculations, errors possible
	Chevalier-Vinh	Transverse isotropic	Very simple documentary results. Possible extension to the viscoelastic region	Approximation of the limiting conditions. Only applicable to transverse isotropic materials
Elasto-dynamic	Behrens	Anything, but but must be periodic	Very simple documentary results. Applicable to a large number of materials	Transposition to the viscoelastic region difficult. Inapplicable to short wave case

Table 2

DIFFERENT VALUES OF THE ELASTIC MODULI OF A SILICON-PHENOLIC RESIN COMPOSITE MATERIALS WITH 63% OF FIBRES (UNITS daN/mm^2)

$\bar{\bar{C}}_{ijk\ell}$	Chevalier-Vinh[17] solution	Pickett solution	Behrens[6] solution	Ultra-sonic measurements Vinh
$\bar{\bar{C}}_{1111}$	1616	1684	1651	1740 ± 30
$\bar{\bar{C}}_{1122}$	718	667	683	756 ± 50
$\bar{\bar{C}}_{1132}$	569	571	569	680 ± 80
$\bar{\bar{C}}_{3333}$	5140	5141	5142	4200 ± 100
$\bar{\bar{C}}_{2323}$	515	519	515	570 ± 12
$\bar{\bar{C}}_{1212}$	449	509	483	492 ± 10

Fibre: $E_2 = 7500\ daN/mm^2$. $\nu_2 = 0.2$

Matrix: $E_1 = 371\ daN/mm^2$. $\nu_1 = 0.34$

Table 3

SOME POSSIBLE METHODS FOR PHYSICAL MEASUREMENTS

Physical methods	Remarks
X rays	
Neutron diffraction	Measurements in K_{ij}
Measurement of specific heat	
Infrared spectroscopy	Transparent bodies Measurements of K_{ij}
Raman spectroscopy	Transparent bodies Measurements of K_{ij}
Ultra- and hyper-sonics	Measurements of $C_{ijk\ell}$ and K_{ij} coupling terms. Coupling between translation and molecular modes
Brillouin diffusion	Crystals. Faraday effect, rotational ability (?) Coupling between the subdued optical mode and acoustic phenomena

For anisotropic, micropolar media the elastic constants are defined by the expression for deformation energy:

$$W = \tfrac{1}{2}C_{ijk\ell}e_{ij}e_{k\ell} + \tfrac{1}{2}K_{ij}(r_i - \psi_i) + \tfrac{1}{2}f_{ijk}e_{ij}(r_k - \psi_k)$$

where e_{ij} = infinitesimal deformation

$r_i = \tfrac{1}{2}e_{imn}(u_{n,m} - u_{m,n})$ = macrorotation

ψ_i = microrotation

e_{imn} = permutation tensor

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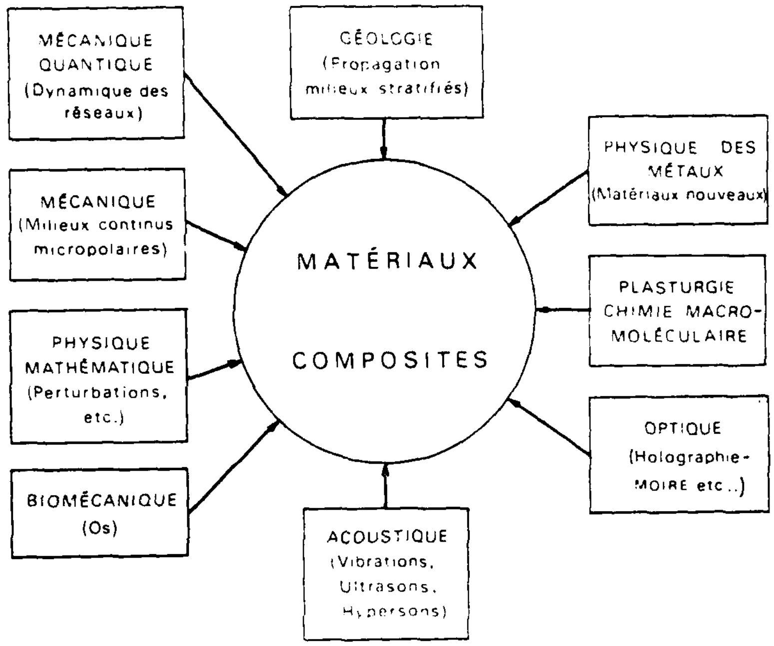
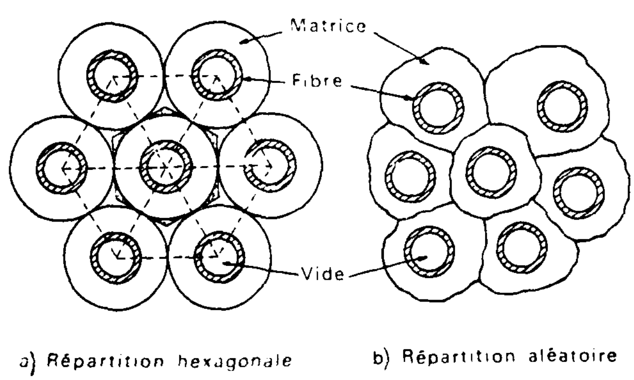


Key:

Matériaux Composites	=	composite materials
Géologie (etc)	=	geology (propagation through stratified media)
Physique des Métaux (etc)	=	metal physics (new materials)
Plasturgie Chimie, etc	=	plastics. Macromolecular chemistry
Optique (etc)	=	optics (Moiré-holography, etc)
Acoustique (etc)	=	acoustics (vibrations, ultrasonics, hypersonics)
Biomécanique (etc)	=	biomechanics (bones)
Physique Mathématique (etc)	=	mathematical physics (disturbances, etc)
Mécanique (etc)	=	mechanics (micropolar continuous media)
Mécanique Quantique (etc)	=	quantum mechanics (network dynamics)

Fig 1 Outline of interdisciplinary studies which affect composite materials



Key:

Matrice	=	matrix
Fibre	=	fibre
Vide	=	empty (space)

Fig 2 Cross-section of a composite material in the plane normal to the fibre axes

(a) hexagonal distribution
(b) random distribution

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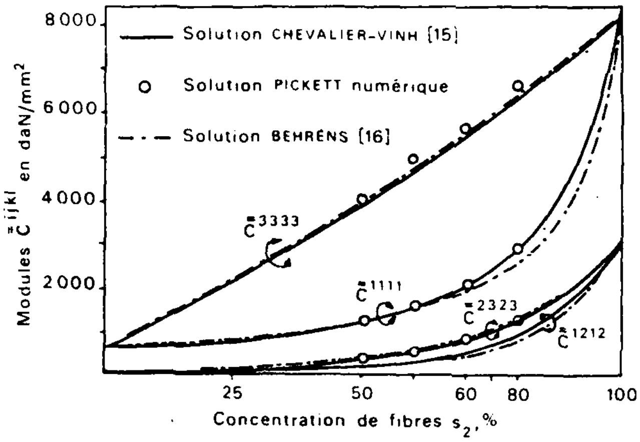
Figs 3&4

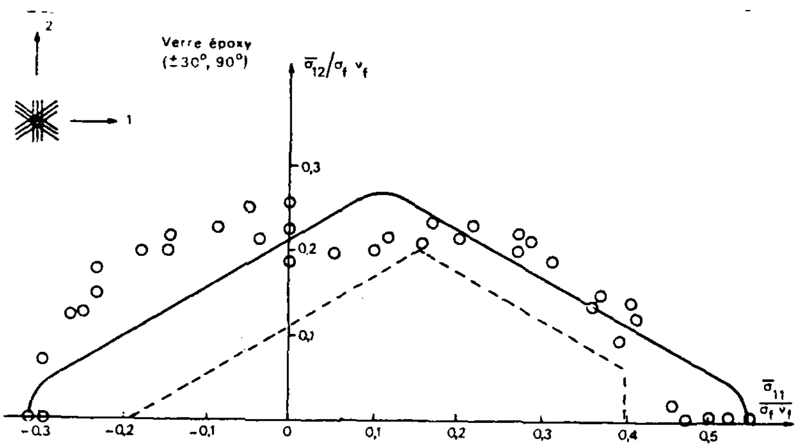


Key:
Modules $\bar{\bar{C}}^{ijkl}$ en daN/mm^2 = $\bar{\bar{C}}^{ijkl}$ modules in daN/mm^2
Concentration de fibres s_2, % = concentration of s_2 fibres, %

———— = Chevalier-Vinh[15] solution
o = Pickett numerical solution
—·—·—·— = Behrens[16] solution

Fig 3 Elastic constants C^{ijkl} of a composite material with hexagonal symmetry

Fibre: E_2 = 7500 daN/mm^2, ν_2 = 0.2
Matrix: E = 371 daN/mm^2, ν_1 = 0.34



Key:
Verre époxy = glass epoxy

o = experimental points
———— = upper limit
------- = lower limit

Fig 4 Determination of the limiting surface of a multi-layer glass-epoxy composite material by limit analysis

The index f refers to the fibre from Le Nizerhy[23]

The errors between theory and experiment are due to manufacturing imperfections and to the influence of interfacial adherance

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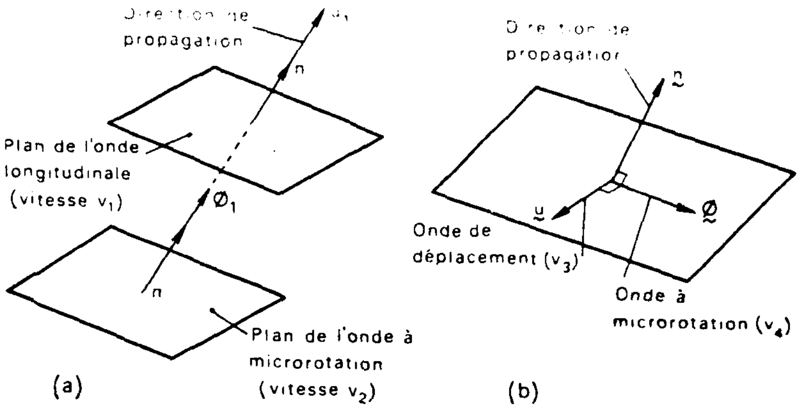


Key:
Direction de propagation = direction of propagation
Plan de l'onde longitudinale (etc) = plan of the longitudinal wave (speed ν_1)
Plan de l'onde à microrotation (etc) = plan of the wave with microrotation (speed ν_2)
Onde de déplacement (ν_3) = displacement wave (ν_3)
Onde à microrotation (ν_4) = wave with microrotation (ν_4)

Fig 5 (a) Waves with longitudinal displacement and waves with microrotation
(b) Coupled transverse waves (speeds ν_3 and ν_4) from Eringen[38]

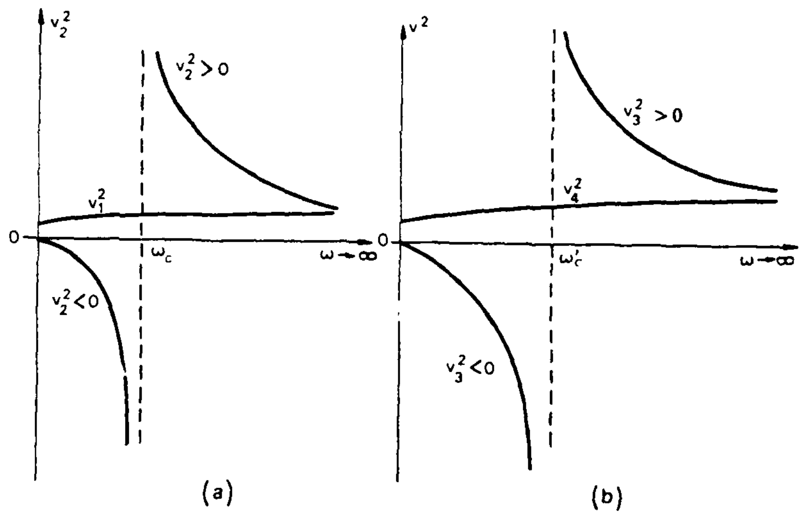


Fig 6 (a) Curve of the square of the speed ν_2^2 (wave with microrotation) as a function of pulse rate;

(b) Curve of the squares of the speeds ν_3^2 and ν_4^2 for transverse waves and waves with microrotation as a function of pulse rate ω. The ω_c and ω_c' pulses can be seen. From Eringen[38].

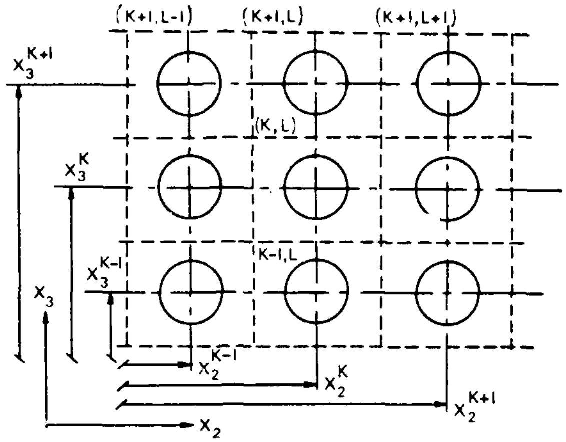
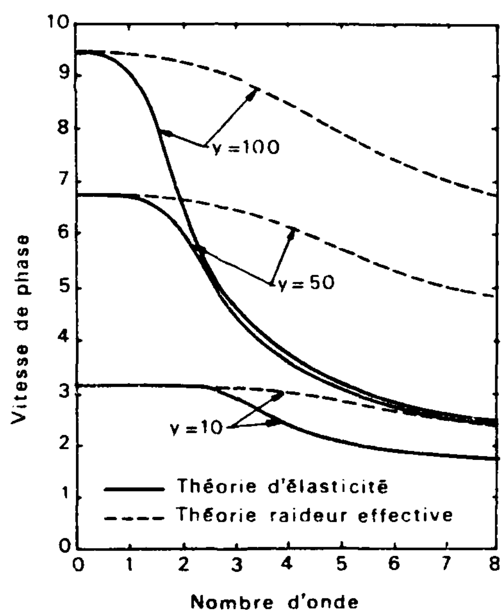


Fig 7 Fibre-reinforced composite material.
Overall and local coordinates



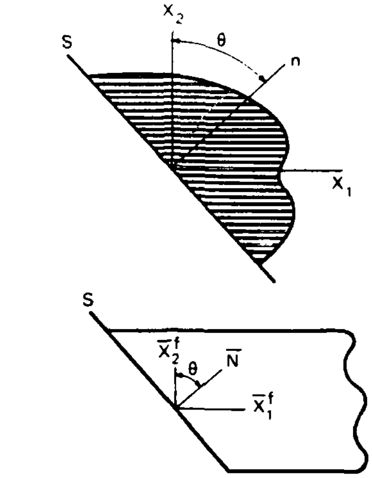
Key:
Vitesse de phase = phase velocity
Nombre d'onde = wave number

———— = theory of elasticity
-------- = effective stiffness theory

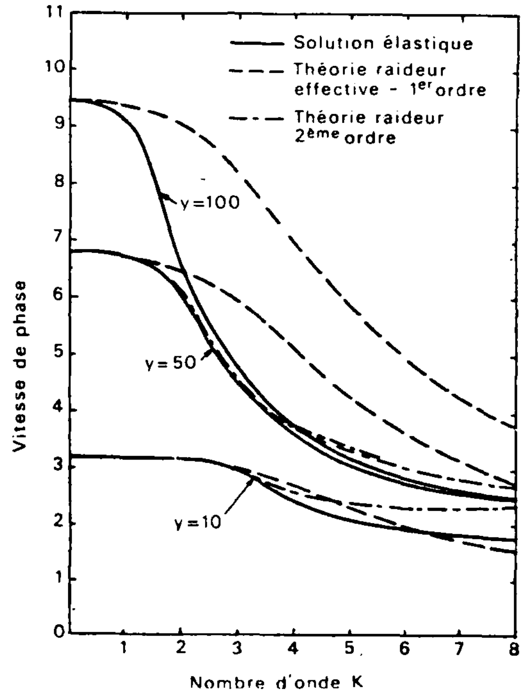
Fig 8 Comparison between the theory of elasticity and the theory of effective stiffness. Longitudinal wave in the direction of the layers. From Sun and Achenbach[39] $y = \mu_f/\mu_m$

Fig 9

(a)



(b)

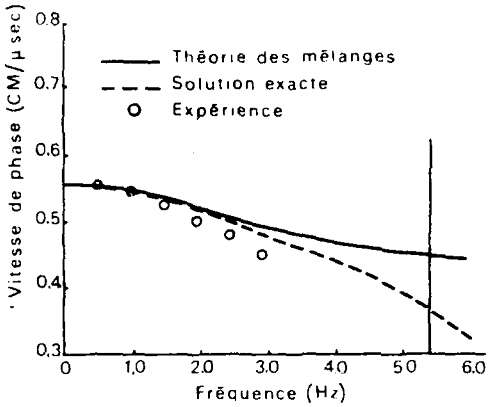


Key:
Vitesse de phase = phase velocity
Nombre d'onde K = wave number

——— = elastic solution
------- = 1st order effective stiffness theory
—·—·— = 2nd order stiffness theory

Fig 9 (a) System of coordinates at the boundaries for a multi-layer composite material

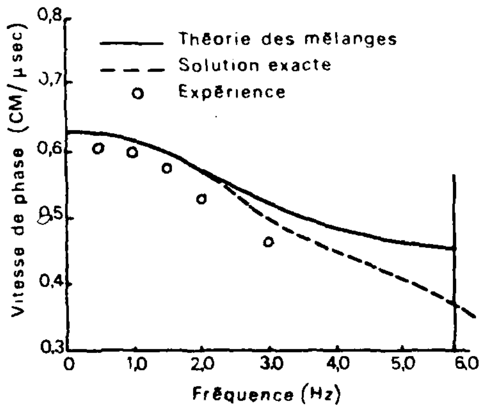
(b) Comparison between the three theories of elasticity and 1st and 2nd order of effective stiffness. From Drumheller[41]; $y = \mu_f/\mu_m$



Key:
Vitesse de phase = phase velocity (cm/μs)
Fréquence = frequency (Hz)

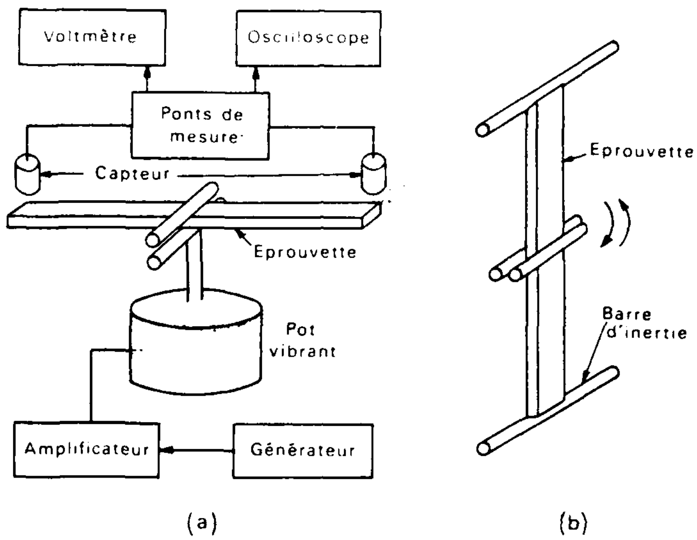
——————— = theory of mixtures
--------- = exact solution
o = experiment

Fig 10 Phase velocity as a function of frequency. Carbon-phenolic composite material. From Drumheller[41]



Key:
(see Fig 10)

Fig 11 Phase velocity as a function of frequency. Boron-phenolic composite material. From Drumheller[41]



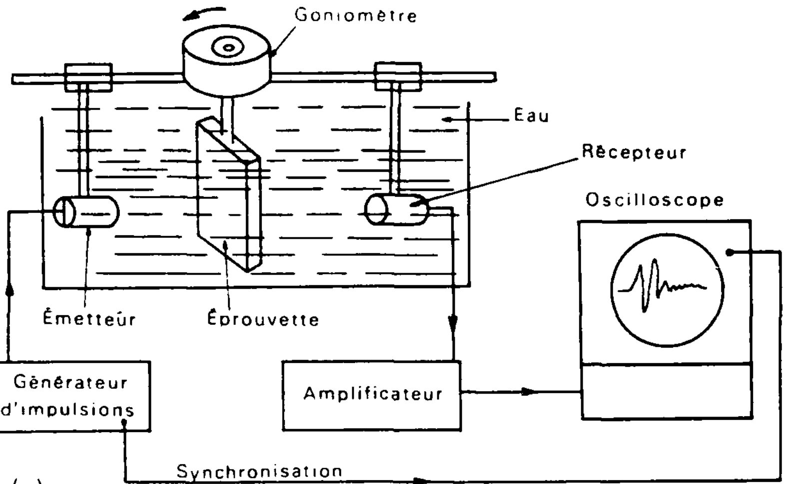
Key:

Voltmètre	= voltmeter
Oscilloscope	= oscilloscope
Ponts de mesure	= measuring bridges
Capteur	= pick-off
Éprouvette	= sample
Pot vibrant	= vibrating pot
Amplificateur	= amplifier
Générateur	= generator
Barre d'inertie	= inertia bar

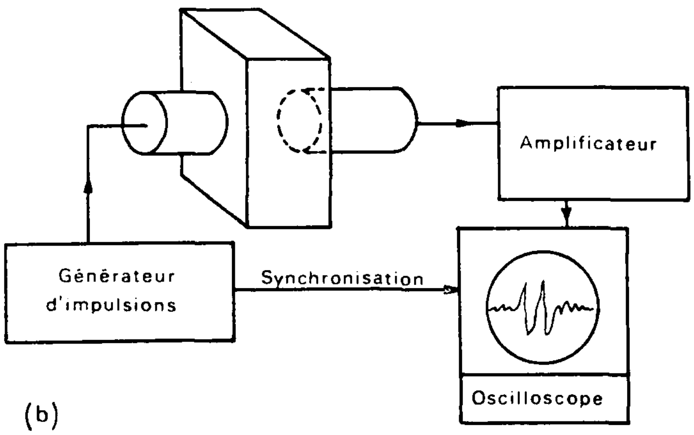
Fig 12 (a) Sketch of the principle of an elasticimeter with alternate bending, for the determination of the complex Young's modulus of composite materials.

(b) The sample is excited into alternate bending by a vibrating pot. The torsion bars at the ends facilitate the display of the vibrations. Determination of Coulomb moduli. The apparatus is similar to that shown in Fig 12(a)

Fig 13



(a)



(b)

Key:
Goniomètre = goniometer
Eau = water
Récepteur = receiver
Oscilloscope = oscilloscope
Émetteur = transmitter
Éprouvette = sample
Générateur d'impulsions = impulse generator
Amplificateur = amplifier
Synchronisation = synchronisation

Fig 13 Principles of measuring elastic constants by means of progressive ultrasonic waves

(a) Immersion apparatus. The sample is fixed to a goniometer. Either longitudinal waves or decoupled transverse waves are obtained.

(b) Direct contact apparatus. The decoupled L or T waves are obtained from transducers. Measurement of transit times

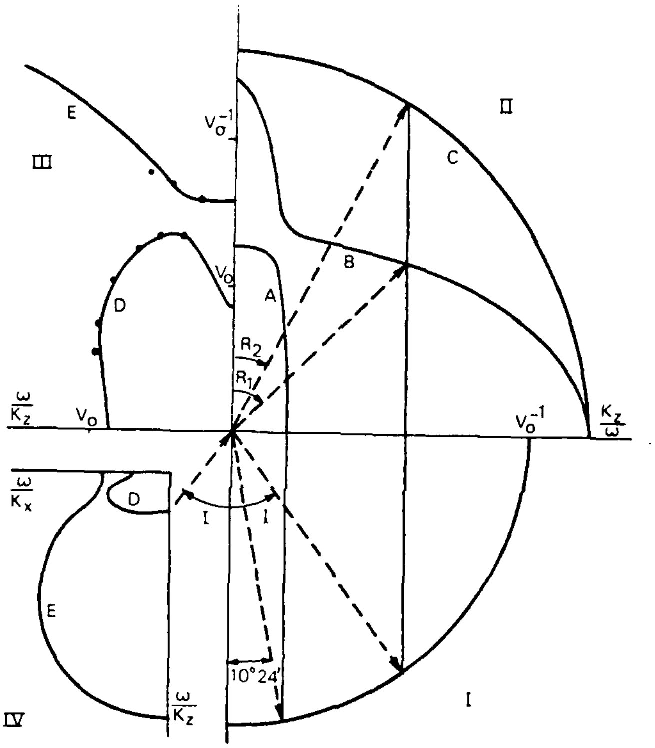
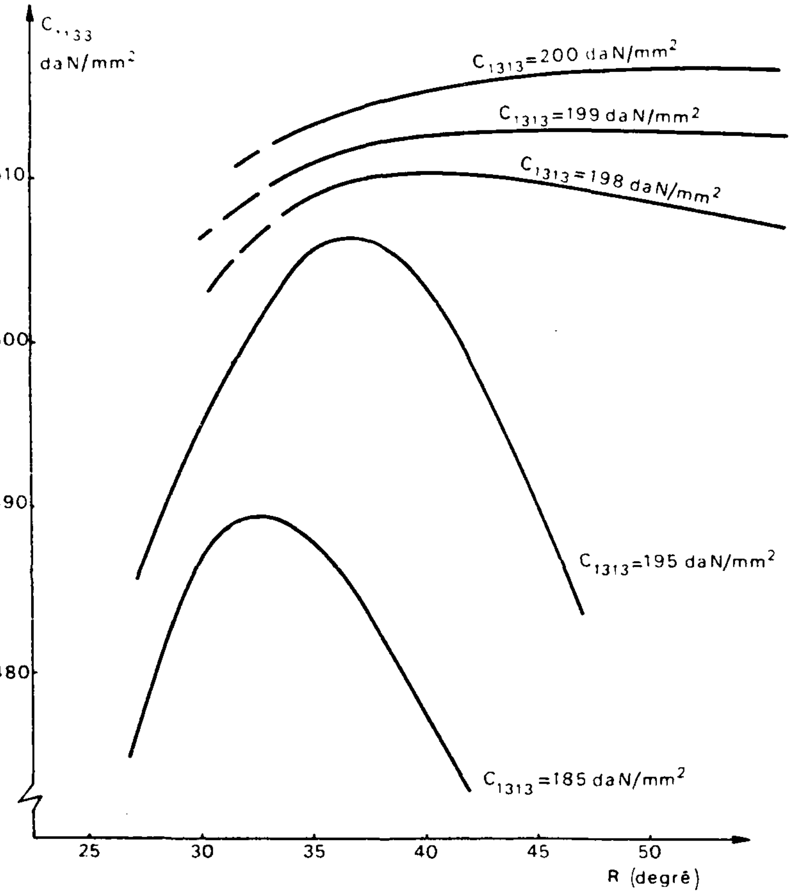


Fig 14 Slowness (inverse of speeds) diagram for the ultrasonic wave across a water-composite material interface, for a Kevlar PRD 49 composite material

Area I: water - wave L
Area II: composite material. Curve A - quasi-longitudinal wave
Curves B and C: quasi-transverse waves (only curve B can be measured)
Curves D and E: radiation diagrams (velocity vectors) for quasi-transverse waves

V_0 is the velocity in water, k_x and k_z the numbers of waves in the x and z directions[59].

Fig 15



Key:
Degré = degrees

Fig 15 The use of a coupled ultrasonic wave to determine C_{1133}. Kevlar composite material. C_{1133} curve as a function of the orientation R taken from Garceau, Vinh[59]. The error perpetrated in C_{1313} shows in the value of C_{1133}. The stationary portion of the curve is adopted for the calculation of C_{1133}

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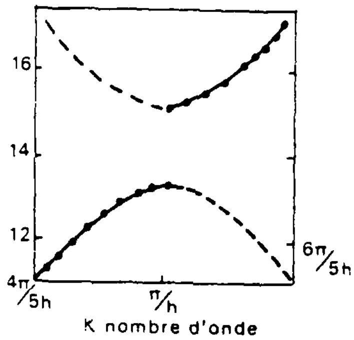
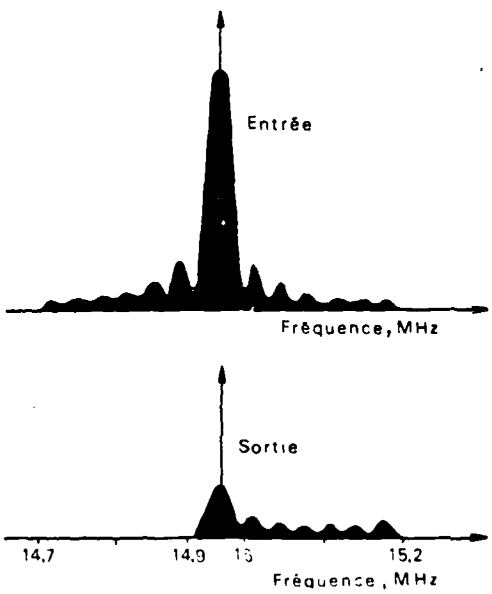


Key:
Nombre d'onde = wave number

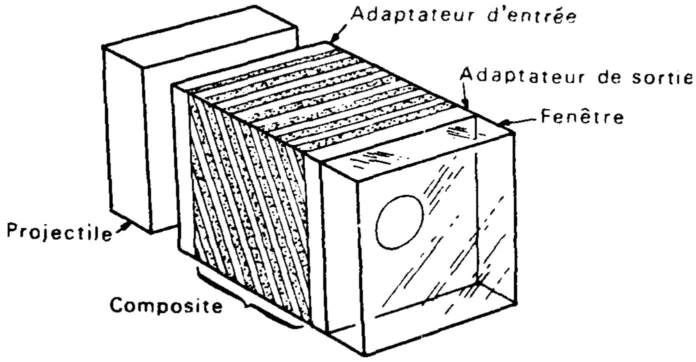
Transverse wave:
------ theory
● experiment

Fig 16 Comparison between the theoretical and experimental dispersions. From Robinson[61]. The pass-band regions can be seen



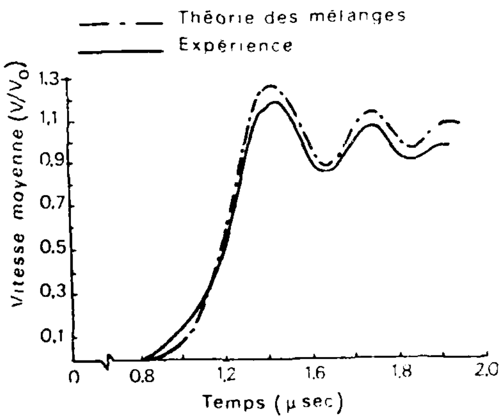
Key:
Entrée = input
Fréquence = frequency, MHz
Sortie = output

Fig 17 Fourier transform spectral analysis of the input and output pulses, from Robinson[61]



Key:
Adaptateur d'entrée = input adapter
Adaptateur de sortie = output adapter
Fenêtre = window
Projectile = projectile
Composite = composite material

Fig 18 Displacement measurement (in a shock test) by interferometry



Key:
Vitesse moyenne = mean velocity (V/V_0)
Temps = time (μs)

—.—.—.— theory of mixtures
——— experiment

Fig 19 Shock tests on a carbon-phenolic composite material. Comparison with the Hegemier and Nayfeh[44,45] theory of mixtures

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